HOLIDAY HOMEWORK

CHAPTER - MATRICES

CLASS - XII

1. If I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and C = $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, show that $(aI + bC)^3 = a^3I + 3a^2bC$.

2. If A = $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, prove that $A^3 - 6A^2 + 7A + 2I = 0$. Hence, find A^{-1} .

3. If f(x) = $\begin{bmatrix} cosx & -sinx & 0 \\ sinx & cosx & 0 \\ 0 & 0 & 1 \end{bmatrix}$, show that f(x) f(y) = f(x + y) .

4. If A = $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then prove that $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ where n is any positive integer .

5. If A = $\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$ and I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then find k so that $A^2 = 8A + kI$.

6. If A = $\begin{bmatrix} cos\theta & isin\theta \\ isin\theta & cos\theta \end{bmatrix}$ then prove by principle of mathematical induction that $A^n = \begin{bmatrix} \cos n\theta & isin\, n\theta \\ isin\, n\theta & \cos n\theta \end{bmatrix}$,

   Where $n \epsilon N$

7. If matrix A = $\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$ where a, b, c are real positive numbers, abc = 1 and $A^TA = I$, then find the value

   of $a^3 + b^3 + c^3$.

8. Express the matrix $\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix .

9. If A = $\begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ find $(AB)^{-1}$.

10. If A = $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$, prove that $A^{-1} = A^2 - 6A + 11I$.

11. For the matrix A = $\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, find the numbers a and b such that $A^2 + aA + bI = O$. Hence find A^{-1} .

12. Solve the following system of equations by matrix method 5x + 3y + z = 16, 2x + y + 3z = 19, x + 2y + 4z = 25.

13. If A = $\begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$, find A^{-1} . Using A^{-1} , solve the system of linear equations: x – 2y = 10, 2x + y + 3z = 8 ,

    -2y + z = 7 .

14. Using matrix method , solve equations for x, y and z : $\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10$; $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10$; $\frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13$.

    [$x \neq 0, y \neq 0, z \neq 0$]

15. If A = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, are two square matrices, find AB and hence solve the system of equations: x – y = 3; 2x + 3y + 4z = 17; y + 2z = 7.

16. Solve the equations by matrix method: 5x + 3y + 7z = 4; 3x + 26y + 2z = 9; 7x + 2y + 10z = 5 .

17. For what value of k, do the equations: 2x – 3y + 2z = a; 5x + 4y – 2z = -3; x – 13y + kz = 9 not have a Unique solution ?

18. Find the adjoint of the following matrix and verify that A .(adj A) = (adj A) . A = $|A|$. I

A = $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$.

19. Two schools A and B want to award their selected students on the values of sincerity, truthfulness and Helpfulness. The school A wants to award Rs x each Rs y each and Rs z each for the three respective values to 3, 2 and 1 students respectively with a total award money at Rs 1600 . School B wants to spend Rs 2300 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as before). If the total amount of award for one prize on each value is Rs 900, using matrices, find the award money for each value.

Apart from these three values suggest one more value which should be considered for award.

20. For the matrix A = $\begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$, find x and y so that $A^2 + xI = yA$. Hence find A^{-1}.

21. Find the adjoint of the matrix A = $\begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ and hence show that A(adj A) = $|A| I_3$.

22. If A = $\begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, show that $A^T A^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$.

23. Solve the following system of equations: 3x + y + z = 1 ; 2x + 2z = 0 ; 5x + y + 2z = 2 .

Using elementary operations, find the inverse of the following matrix :

24. $\begin{bmatrix} 10 & -2 \\ -5 & 1 \end{bmatrix}$

25. $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

26. $\begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

27. $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$