



HOLIDAY HOMEWORK
CHAPTER - DIFFERENTIATION
CLASS - XII

Differentiate the following functions w.r.t. x:

1. If $Y = \sin^5 x \sin x^5$.
2. If $y = \sqrt{\frac{1-\sin 2x}{1+\sin 2x}}$, show that $dy/dx + \sec^2\left(\frac{\pi}{4} - x\right) = 0$
3. If $y = \log \sqrt{\frac{1+x \cos x}{1+x \cos x}}$, find dy/dx in simplified form.
4. If $y = (x + \sqrt{x^2 + a^2})^n$ prove that $\frac{dy}{dx} = \frac{ny}{\sqrt{x^2+a^2}}$
5. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ ($x \neq y$), then prove that : $dy/dx = -\frac{1}{(1+x)^2}$.
6. Differentiate $\tan^2 x$ w. r. t. $\cos^2 x$.
7. Differentiate $\sqrt{\frac{1+e^x}{1-e^x}}$.
8. If $y = \sqrt{x^2 + 1} - \log\left(\frac{1}{x} + \sqrt{1 + \frac{1}{x^2}}\right)$, find dy/dx .
9. Differentiate $\tan^{-1} \frac{\sqrt{1+x^2}-1}{x}$
10. Differentiate $\tan^{-1} \left(\frac{a \cos x - b \sin x}{b \cos x + a \sin x}\right)$.
11. If $y = \sin^{-1} [x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}]$, find $\frac{dy}{dx}$.
12. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then prove that $dy/dx = \sqrt{\frac{1-y^2}{1-x^2}}$.
13. If $y = \sin^{-1} \frac{1}{\sqrt{1+x^2}} + \tan^{-1} \frac{\sqrt{(1+x^2)}-1}{x}$, then show that $dy/dx = \frac{-1}{2(1+x^2)}$
14. Differentiate $\tan^{-1} \frac{x}{1+\sqrt{1-x^2}}$ w.r.t. $\sin\left(2 \cot^{-1} \sqrt{\frac{1+x}{1-x}}\right)$.
15. Differentiate $\tan^{-1} \left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$ w.r.t. $\sin^{-1} \left(\frac{2x}{1+x^2}\right)$.
16. If $\log \sqrt{x^2 + y^2} = \tan^{-1} \frac{y}{x}$, find dy/dx
17. If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dots \text{to } \infty}}}$, prove that $(2y-1) dy/dx = \cos x$.
18. If $\sin y = x \sin(a+y)$, prove that $dy/dx = \frac{\sin^2(a+y)}{\sin a}$.
19. Find dy/dx when $x = a\left(\cos t + \log \tan \frac{t}{2}\right)$, $y = a \sin t$.
20. Differentiate : $x^{\cos x} + \frac{x^2+1}{x^2-1}$.
21. If $y^x + x^y + x^x = a^b$, find dy/dx .
22. Find dy/dx when $x^m y^n = (x+y)^{m+n}$
23. If $y = (\cos x)^{(\cos x)^{(\cos x) \dots \text{to } \infty}}$, prove that $dy/dx = \frac{-y^2 \tan x}{1-y \log \cos x}$.
24. If $y = a^{x a^{x a^{x \dots \infty}}}$, prove that $dy/dx = \frac{y^2 \log y}{x[1-y \log x \log y]}$.
25. If $x = a \sin 2t (1 + \cos 2t)$ and $y = b \cos 2t (1 - \cos 2t)$, show that $\left(\frac{dy}{dx}\right)_{t=\frac{\pi}{4}} = b/a$

26. Find dy/dx , if $y = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$.

27. If $y = (\cot^{-1}x)^2$, Show that $(1+x^2)^2 \frac{d^2y}{dx^2} + 2x(1+x^2) \frac{dy}{dx} = 2$.

28. If $\log y = \tan^{-1}x$, prove that $(1+x^2) \frac{d^2y}{dx^2} + (2x+1) \frac{dy}{dx} = 0$.

29. If $y = \frac{x \sin^{-1}x}{\sqrt{1-x^2}}$, prove that $(1-x^2) \frac{dy}{dx} = x + \frac{y}{x}$.

30. If $y = \sin(m \sin^{-1}x)$, show that $(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - m^2 y = 0$.